Real Numbers

Practice set 2.1

Q. 1. Classify the decimal form of the given rational numbers into terminating and non-terminating recurring type

i. $\frac{13}{5}$ ii. $\frac{2}{11}$

iii. $\frac{29}{16}$ iv. $\frac{17}{125}$

v. $\frac{11}{6}$

Answer : i.

$\frac{13}{5} = 2.6$

∵ The division is exact

∴ it is a terminating decimal.

ii.

$\frac{2}{11} = 0.181818 \ldots..$

∵ The division never ends and the digits '18' is repeated endlessly

∴ it is a non-terminating recurring type decimal.

iii.

$\frac{29}{16} = 1.8125$

∵ The division is exact

∴ it is a terminating decimal.

iv.

$\frac{17}{125} = 0.136$

∵ The division is exact

∴ it is a terminating decimal.

v.

$\frac{11}{6} = 1.83333 \ldots..$

∵ The division never ends and the digit '3' is repeated endlessly

∴ it is a non-terminating recurring type decimal.

Q. 2. Write the following rational numbers in decimal form.

i. $\frac{127}{200}$ **ii.** $\frac{25}{99}$

iii. $\frac{23}{7}$ **iv.** $\frac{4}{5}$

v. $\frac{17}{8}$

Answer :

i. $\frac{127}{200} = 0.635$

ii. $\frac{25}{99} = 0.252525 \ldots.$

iii. $\frac{23}{7} = 3.285714285714285714 \ldots \ldots..$

iv. $\frac{4}{5} = 0.8$

v. $\frac{17}{8} = 2.125$

Q. 3. Write the following rational numbers in form.

i. 0.6^{*} **ii.** $0.\overline{37}$

iii. $3.\overline{17}$ **iv.** $15.\overline{89}$

v. $2.\overline{514}$

Answer :

i. $0.\dot{6}$

Let $x = 0.\dot{6} = 0.6666..$

⇒ 10x = 6.66666......

Now,

10x - x = 6.66 – 0.6666

⇒9x = 6

$\Rightarrow x = \frac{6}{9}$

$\Rightarrow 0.\dot{6} = \frac{6}{9} = \frac{2}{3}$

ii. $0.\overline{37}$

Let $x = 0.\overline{37} = 0.3737 ...$

⇒ 100x = 37.3737......

Now,

100x - x = 37.3737 – 0.3737

⇒ 99x = 37

$\Rightarrow x = \frac{37}{99}$

$\Rightarrow 0.\overline{37} = \frac{37}{99}$

iii. $3.\overline{17}$

Let $x = 3.\overline{17} = 3.1717 \ldots$

⇒ 100x = 317.1717......

Now,

100x - x = 317.1717 – 3.1717

⇒ 99x = 314

$\Rightarrow x = \frac{314}{99}$

$\Rightarrow 3.\overline{17} = \frac{314}{99}$

iv. $15.\overline{89}$

Let $x = 15.\overline{89} = 15.8989 \ldots$

⇒ 100x = 1589.8989......

Now,

100x - x = 1589.8989 – 15.8989

⇒ 99x = 1574

$\Rightarrow x = \frac{1574}{99}$

$\Rightarrow 15.\overline{89} = \frac{1574}{99}$

v. $2.\overline{514}$

Let $x = 2.\overline{514} = 2.514514\ldots$

⇒ 1000x = 2514.514514......

Now,

1000x - x = 2514.514514 – 2.514514

⇒ 999x = 2512

$\Rightarrow x = \frac{2512}{999}$

$\Rightarrow 2.\overline{514} = \frac{2512}{999}$

Practice set 2.1

Q. 1. Show that is 4√2 an irrational number.

Answer : Let us assume that 4√2 is a rational number

$\therefore\ 4\sqrt{2} = \frac{a}{b}$

where, b≠0 and a, b are integers

$\Rightarrow\ \sqrt{2} = \frac{a}{4b}$

∵ a, b are integers ∴ 4b is also integer

$\Rightarrow \frac{a}{4b}$ is rational which cannot be possible

$\because \frac{a}{4b} = \sqrt{2}$ which is an irrational number

∵ it is contradicting our assumption

∴ the assumption was wrong

Hence, $4\sqrt{2}$ is an irrational number

Q. 2. Prove that 3 + √5 is an irrational number.

Answer : Let us assume that $3 + \sqrt{5}$ is a rational number

$$\therefore\ 3 + \sqrt{5} = \frac{a}{b}$$

where, b≠0 and a, b are integers

$$\Rightarrow\ \sqrt{5} = \frac{a}{b} - 3$$

$$\Rightarrow\ \sqrt{5} = \frac{a - 3b}{b}$$

∵ a, b are integers ∴ a – 3b is also integer

$\Rightarrow\ \frac{a-3b}{b}$ is rational which cannot be possible

$\because \frac{a-3b}{b} = \sqrt{5}$ which is an irrational number

∵ it is contradicting our assumption ∴ the assumption was wrong

Hence, $3 + \sqrt{5}$ is an irrational number

Q. 3. Represent the numbers √5 and √10 on a number line.

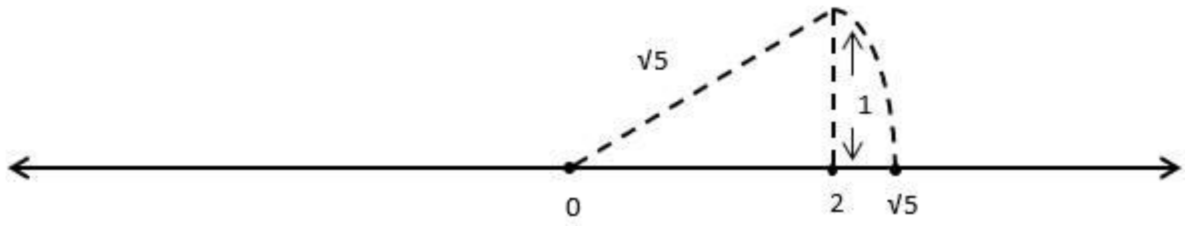
Answer : By Pythagoras theorem,

$(\sqrt{5})^2 = 2^2 + 1^2$

$\Rightarrow (\sqrt{5})^2 = 4 + 1$

$$\Rightarrow\ \sqrt{5} = \sqrt{4 + 1}$$

First mark 0 and 2 on the number line. Then, draw a perpendicular of 1 unit from 2. And Join the top of perpendicular and 0. This line would be equal to √5. Now measure the line with compass and marc an arc on the number line with the same measurement. This point is √5.



Also,

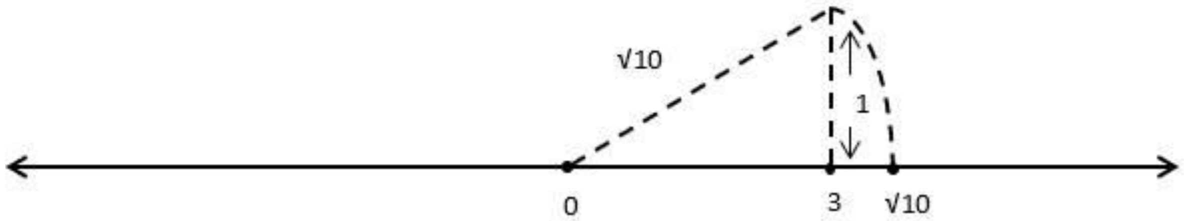
By Pythagoras theorem,

$(\sqrt{10})^2 = 3^2 + 1^2$

$\Rightarrow (\sqrt{10})^2 = 9 + 1$

$\Rightarrow \sqrt{10} = \sqrt{9 + 1}$

First mark 0 and 3 on the number line. Then, draw a perpendicular of 1 unit from 3. And Join the top of perpendicular and 0. This line would be equal to √10. Now measure the line with compass and marc an arc on the number line with the same measurement. This point is √10.



Q. 4 A. Write any three rational numbers between the two numbers given below.

0.3 and -0.5

Answer : 0.3 and -0.5

To find a rational number x between two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use

$$x = \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right)$$

Therefore, to find rational number x (let) between

$0.3 = \frac{3}{10}$ and $-0.5 = \frac{-5}{10}$

$$x = \frac{1}{2}\left(\frac{3}{10} + \frac{-5}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \left(\frac{3-5}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \frac{-2}{10}$$

$$\Rightarrow x = \frac{-1}{10} = -0.1$$

Now if we find a rational number between $\frac{3}{10}$ and $\frac{-1}{10}$ it will also be between 0.3 and -0.5 since $\frac{-1}{10}$ lies between 0.3 and -0.5.

Therefore, to find rational number y (let) between $\frac{3}{10}$ and $\frac{-1}{10}$

$$y = \frac{1}{2}\left(\frac{3}{10} + \frac{-1}{10}\right)$$

$$\Rightarrow y = \frac{1}{2}\left(\frac{3-1}{10}\right)$$

$$\Rightarrow y = \frac{1}{2} \times \frac{2}{10}$$

$$\Rightarrow y = \frac{1}{10} = 0.1$$

Now if we find a rational number between $\frac{-1}{10}$ and $\frac{1}{10}$ it will also be between 0.3 and -0.5 since $\frac{1}{10}$ lies between 0.3 and -0.5.

Therefore, to find rational number z (let) between $\frac{1}{10}$ and $\frac{-5}{10}$.

$$z = \frac{1}{2}\left(\frac{1}{10} + \frac{-5}{10}\right)$$

$$\Rightarrow z = \frac{1}{2}(\frac{1-5}{10})$$

$$\Rightarrow z = \frac{1}{2} \times \frac{-4}{10}$$

$$\Rightarrow z = \frac{-2}{10} = -0.2$$

Hence the numbers are -0.2, -0.1 and 0.1

Q. 4 B. Write any three rational numbers between the two numbers given below.

-2.3 and -2.33

Answer : -2.3 and -2.33

To find a rational number x between two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use

$$x = \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right)$$

Therefore, to find rational number x (let) between $-2.3 = \frac{-23}{10}$ and $-2.33 = \frac{-233}{100}$

$$x = \frac{1}{2}\left(\frac{-23}{10} + \frac{-233}{100}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \left(\frac{-230-233}{100}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \frac{-463}{100}$$

⇒ x = -2.315

Now if we find a rational number between $-2.315 = \frac{-2315}{1000}$ and $-2.3 = \frac{-23}{10}$ it will also be between -2.3 and -2.33 since -2.315 lies between -2.3 and -2.33

Therefore, to find rational number y (let) between $\frac{-2315}{1000}$ and $\frac{-23}{10}$

$$y = \frac{1}{2}\left(\frac{-2315}{1000} + \frac{-23}{10}\right)$$

$$\Rightarrow y = \frac{1}{2}\left(\frac{-2315 - 2300}{1000}\right)$$

$$\Rightarrow y = \frac{1}{2} \times \frac{-4615}{1000}$$

⇒ y = -2.3075

Now if we find a rational number between $-2.315 = \frac{-2315}{1000}$ and $-2.33 = \frac{-233}{100}$ it will also be between -2.3 and -2.33 since -2.315 lies between -2.3 and -2.33

Therefore, to find rational number z (let) between $\frac{-2315}{1000}$ and $\frac{-233}{100}$

$$z = \frac{1}{2}\left(\frac{-2315}{1000} + \frac{-233}{100}\right)$$

$$\Rightarrow z = \frac{1}{2}\left(\frac{-2315 - 2330}{1000}\right)$$

$$\Rightarrow z = \frac{1}{2} \times \frac{-4645}{1000}$$

⇒ z = -2.3225

Hence the numbers are -2.3225, -2.3075 and -2.315

Q. 4 C. Write any three rational numbers between the two numbers given below.

5.2 and 5.3

Answer : 5.2 and 5.3

To find a rational number x between two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use

$$x = \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right)$$

Therefore, to find rational number x (let) between $5.2 = \frac{52}{10}$ and $5.3 = \frac{53}{10}$

$$x = \frac{1}{2}\left(\frac{52}{10} + \frac{53}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \left(\frac{52 + 53}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \frac{105}{10}$$

⇒ x = 5.25

Now if we find a rational number between $5.25 = \frac{525}{100}$ and $5.2 = \frac{52}{10}$ it will also be between 5.2 and 5.3 since 5.25 lies between 5.2 and 5.3

Therefore, to find rational number y (let) between $5.25 = \frac{525}{100}$ and $5.2 = \frac{52}{10}$

$$y = \frac{1}{2}\left(\frac{525}{100} + \frac{52}{10}\right)$$

$$\Rightarrow y = \frac{1}{2}\left(\frac{525 + 520}{100}\right)$$

$$\Rightarrow y = \frac{1}{2} \times \frac{1045}{100}$$

⇒ y = 5.225

Now if we find a rational number between $5.25 = \frac{525}{100}$ and $5.3 = \frac{53}{10}$ it will also be between 5.2 and 5.3 since 5.25 lies between 5.2 and 5.3

Therefore, to find rational number z (let) between $5.25 = \frac{525}{100}$ and $5.3 = \frac{53}{10}$

$$z = \frac{1}{2}\left(\frac{525}{100} + \frac{53}{10}\right)$$

$$\Rightarrow z = \frac{1}{2}\left(\frac{525 + 530}{100}\right)$$

$$\Rightarrow z = \frac{1}{2} \times \frac{1055}{100}$$

⇒ z = 5.275

Hence the numbers are 5.225, 5.25 and 5.275

Q. 4 D. Write any three rational numbers between the two numbers given below.

-4.5 and 4.6

Answer : -4.5 and 4.6

To find a rational number x between two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use

$$x = \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right)$$

Therefore, to find rational number x (let) between $-4.5 = \frac{-45}{10}$ and $4.6 = \frac{46}{10}$

$$x = \frac{1}{2}\left(\frac{-45}{10} + \frac{46}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \left(\frac{-45 + 46}{10}\right)$$

$$\Rightarrow x = \frac{1}{2} \times \frac{1}{10}$$

⇒ x = 0.05

Now if we find a rational number between $-4.5 = \frac{-45}{10}$ and $0.05 = \frac{5}{100}$ it will also be between -4.5 and 4.6 since 0.05 lies between -4.5 and 4.6

Therefore, to find rational number y (let) between $-4.5 = \frac{-45}{10}$ and $0.05 = \frac{5}{100}$

$$y = \frac{1}{2}\left(\frac{-45}{10} + \frac{5}{100}\right)$$

$$\Rightarrow y = \frac{1}{2}\left(\frac{-450 + 5}{100}\right)$$

$$\Rightarrow y = \frac{1}{2} \times \frac{-445}{100}$$

$\Rightarrow$ y = -2.225

Now if we find a rational number between $4.6 = \frac{46}{10}$ and $0.05 = \frac{5}{100}$ it will also be between -4.5 and 4.6 since 0.05 lies between -4.5 and 4.6

Therefore, to find rational number z (let) between $4.6 = \frac{46}{10}$ and $0.05 = \frac{5}{100}$

$$z = \frac{1}{2}\left(\frac{46}{10} + \frac{5}{100}\right)$$

$$\Rightarrow z = \frac{1}{2}\left(\frac{460+5}{100}\right)$$

$$\Rightarrow z = \frac{1}{2} \times \frac{465}{100}$$

$\Rightarrow$ z = 2.325

Hence the numbers are -2.225, 0.05and 2.325

Practice set 2.3

Q. 1. State the order of the surds given below.

i. $\sqrt[3]{7}$ **ii.** $5\sqrt{12}$

iii. $\sqrt[4]{10}$ **iv.** $\sqrt{39}$

v. $\sqrt[3]{18}$

Answer : In $\sqrt[n]{a}$, n is called the order of the surd.

Therefore,

i. $\sqrt[3]{7}$

In this, the order of surd is 3.

ii. $\sqrt[5]{12}$

In this, the order of surd is 5.

iii. $\sqrt[4]{10}$

In this, the order of surd is 4.

iv. $\sqrt{39}$

In this, the order of surd is 2.

v. $\sqrt[3]{18}$

In this, the order of surd is 3

Q. 2. State which of the following are surds. Justify.

i. $\sqrt[3]{51}$ **ii.** $\sqrt[4]{51}$
iii. $\sqrt[5]{81}$ **iv.** $\sqrt{256}$
v. $\sqrt[3]{64}$ **vi.** $\sqrt{\frac{22}{7}}$

Answer : Surds are numbers left in root form (√) to express its exact value. It has an infinite number of non-recurring decimals. Therefore, surds are irrational numbers.

Therefore,

i. $\sqrt[3]{51}$

It is a surd ∵ it cannot be expressed as a rational number.

ii. $\sqrt[4]{51}$

It is a surd ∵ it cannot be expressed as a rational number.

iii. $\sqrt[5]{81} = \sqrt[5]{3^4}$

It is a surd ∵ it cannot be expressed as a rational number.

iv. √256 = √162 = 16

It is not a surd ∵ it is a rational number.

v. $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$

It is not a surd ∵ it is a rational number.

vi. $\sqrt{\frac{22}{7}}$

It is a surd ∵ it cannot be expressed as a rational number.

Q. 3. Classify the given pair of surds into like surds and unlike surds.

i. √52, 5√13

ii. √68, 5√3

iii. 4√18, 7√2

iv. 19√12, 6√3

v. 5√22, 7√33

vi. 5√5, √75

Answer : Two or more surds are said to be similar or like surds if they have the same surd-factor.

And,

Two or more surds are said to be dissimilar or unlike when they are not similar.

Therefore,

i. √52, 5√13

√52 = √(2×2×13) = 2√13

5√13

∵ both surds have same surd-factor i.e., √13.

∴ they are like surds.

ii. √68, 5√3

√68 = √(2×2×17) = 2√17

5√3

∵ both surds have different surd-factors √17 and √3.

∴ they are unlike surds.

iii. 4√18, 7√2

4√18 = 4√(2×3×3) = 4×3√2 = 12√2

7√2

∵ both surds have same surd-factor i.e., √2.

∴ they are like surds.

iv. 19√12, 6√3

19√12 = 19√(2×2×3) = 19×2√3 = 38√3

6√3

∵ both surds have same surd-factor i.e., √3.

∴ they are like surds.

v. 5√22, 7√33

∵ both surds have different surd-factors √22 and √33.

∴ they are unlike surds.

vi. 5√5, √75

5√5

√75 = √(5×5×3) = 5√3

∵ both surds have different surd-factors √5 and √3.

∴ they are unlike surds.

Q. 4. Simplify the following surds.

i. √27
ii. √50
iii. √250
iv. √112
v. √168

Answer : i. $\sqrt{27} = \sqrt{3 \times 3 \times 3}$

$\Rightarrow \sqrt{27} = \sqrt{3 \times (3)^2}$

$\Rightarrow \sqrt{27} = 3\sqrt{3}$

ii. $\sqrt{50} = \sqrt{2 \times 5 \times 5}$

$\Rightarrow \sqrt{50} = \sqrt{2 \times (5)^2}$

$\Rightarrow \sqrt{50} = 5\sqrt{2}$

iii. $\sqrt{250} = \sqrt{2 \times 5 \times 5 \times 5}$

$\Rightarrow \sqrt{250} = \sqrt{10 \times (5)^2}$

$\Rightarrow \sqrt{250} = 5\sqrt{10}$

iv. $\sqrt{112} = \sqrt{2 \times 2 \times 2 \times 2 \times 7}$

$\Rightarrow \sqrt{112} = \sqrt{(2)^2 \times (2)^2 \times 7}$

$\Rightarrow \sqrt{112} = 2 \times 2 \times \sqrt{7}$

$\Rightarrow \sqrt{112} = 4\sqrt{7}$

v. $\sqrt{168} = \sqrt{2 \times 2 \times 2 \times 3 \times 7}$

$\Rightarrow \sqrt{112} = \sqrt{(2)^2 \times 2 \times 3 \times 7}$

$\Rightarrow \sqrt{112} = 2 \times \sqrt{42}$

$\Rightarrow \sqrt{112} = 2\sqrt{42}$

Q. 5. Compare the following pair of surds.

i. 7√2, 5√3
ii. √247, √274
iii. 2√7, √28
iv. 5√5, 7√2
v. 4√42, 9√2
vi. 5√3, 9
vii. 7, 2√5

Answer : i. 7√2 , 5√3

$(7\sqrt{2})^2 = 7 \times 7 \times \sqrt{2} \times \sqrt{2}$

$\Rightarrow (7\sqrt{2})^2 = 49 \times 2$

$\Rightarrow (7\sqrt{2})^2 = 98$

And

$(5\sqrt{3})^2 = 5 \times 5 \times \sqrt{3} \times \sqrt{3}$

$\Rightarrow (5\sqrt{3})^2 = 25 \times 3$

$\Rightarrow (5\sqrt{3})^2 = 75$

Clearly,

98 > 75

∴ $7\sqrt{2} > 5\sqrt{3}$

ii. $\sqrt{247}$, $\sqrt{274}$

$(\sqrt{247})^2 = 247$

And

$(\sqrt{274})^2 = 274$

Clearly,

247 < 274

∴ $\sqrt{247} < \sqrt{274}$

iii. $2\sqrt{7}$, $\sqrt{28}$

$(2\sqrt{7})^2 = 2 \times 2 \times \sqrt{7} \times \sqrt{7}$

$\Rightarrow (2\sqrt{7})^2 = 4 \times 7$

$\Rightarrow (2\sqrt{7})^2 = 28$

And

$(\sqrt{28})^2 = 28$

Clearly,

28 = 28

∴ $2\sqrt{7} = \sqrt{28}$

iv. $5\sqrt{5}$, $7\sqrt{2}$

$(5\sqrt{5})^2 = 5 \times 5 \times \sqrt{5} \times \sqrt{5}$

$\Rightarrow (5\sqrt{5})^2 = 25 \times 5$

$\Rightarrow (5\sqrt{5})^2 = 125$

And

$(7\sqrt{2})^2 = 7 \times 7 \times \sqrt{2} \times \sqrt{2}$

$\Rightarrow (7\sqrt{2})^2 = 49 \times 2$

$\Rightarrow (7\sqrt{2})^2 = 98$

Clearly,

125 = 98

$\therefore 5\sqrt{5} = 7\sqrt{2}$

v. $4\sqrt{42}, 9\sqrt{2}$

$(4\sqrt{42})^2 = 4 \times 4 \times \sqrt{42} \times \sqrt{42}$

$\Rightarrow (4\sqrt{42})^2 = 16 \times 42$

$\Rightarrow (4\sqrt{42})^2 = 672$

And

$(9\sqrt{2})^2 = 9 \times 9 \times \sqrt{2} \times \sqrt{2}$

$\Rightarrow (9\sqrt{2})^2 = 81 \times 2$

$\Rightarrow (9\sqrt{2})^2 = 162$

Clearly,

672 > 162

$\therefore 4\sqrt{42} > 9\sqrt{2}$

vi. $5\sqrt{3}, 9$

$(5\sqrt{3})^2 = 5 \times 5 \times \sqrt{3} \times \sqrt{3}$

$\Rightarrow (5\sqrt{3})^2 = 25 \times 3$

$\Rightarrow (5\sqrt{3})^2 = 75$

And

$(9)^2 = 9 \times 9$

$\Rightarrow (9)^2 = 81$

Clearly,

$75 < 81$

$\therefore 5\sqrt{3} < 9$

vii. $7, 2\sqrt{5}$

$(2\sqrt{5})^2 = 2 \times 2 \times \sqrt{5} \times \sqrt{5}$

$\Rightarrow (2\sqrt{5})^2 = 4 \times 5$

$\Rightarrow (2\sqrt{5})^2 = 20$

And

$(7)^2 = 7 \times 7$

$\Rightarrow (7)^2 = 49$

Clearly,

$49 > 20$

$\therefore 7 > 2\sqrt{5}$

Q. 6. Simplify.

i. $5\sqrt{3} + 8\sqrt{3}$
ii. $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$
iii. $7\sqrt{48} - \sqrt{27} - \sqrt{3}$
iv.

$$\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7}$$

Answer : i. $5\sqrt{3} + 8\sqrt{3}$

$5\sqrt{3} + 8\sqrt{3} = (5 + 8)\sqrt{3}$

$\Rightarrow 5\sqrt{3} + 8\sqrt{3} = 13\sqrt{3}$

ii. $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

$9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = 9\sqrt{5} - 4\sqrt{5} + \sqrt{(5 \times 5 \times 5)}$

$\Rightarrow 9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = 9\sqrt{5} - 4\sqrt{5} + \sqrt{(5 \times 5 \times 5)}$

$\Rightarrow 9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = 9\sqrt{5} - 4\sqrt{5} + 5\sqrt{5}$

$\Rightarrow 9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = (9 - 4 + 5)\sqrt{5}$

$\Rightarrow 9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = 10\sqrt{5}$

iii. $7\sqrt{48} - \sqrt{27} - \sqrt{3}$

$7\sqrt{48} - \sqrt{27} - \sqrt{3} = 7\sqrt{(2 \times 2 \times 2 \times 2 \times 3)} - \sqrt{(3 \times 3 \times 3)} - \sqrt{3}$

$\Rightarrow 7\sqrt{48} - \sqrt{27} - \sqrt{3} = 7 \times 4\sqrt{3} - 3\sqrt{3} - \sqrt{3}$

$\Rightarrow 7\sqrt{48} - \sqrt{27} - \sqrt{3} = 28\sqrt{3} - 3\sqrt{3} - \sqrt{3}$

$\Rightarrow 7\sqrt{48} - \sqrt{27} - \sqrt{3} = (28 - 3 - 1)\sqrt{3}$

$\Rightarrow 7\sqrt{48} - \sqrt{27} - \sqrt{3} = 24\sqrt{3}$

iv. $\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7}$

$$\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7} = \left(1 - \frac{3}{5} + 2\right)\sqrt{7}$$

$$\Rightarrow \sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7} = \left(\frac{5 - 3 + 10}{5}\right)\sqrt{7}$$

$$\Rightarrow \sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7} = \frac{12}{5}\sqrt{7}$$

Q. 7. Multiply and write the answer in the simplest form.

i. $3\sqrt{12} \times \sqrt{18}$
ii. $3\sqrt{12} \times 7\sqrt{15}$
iii. $3\sqrt{8} \times \sqrt{5}$
iv. $5\sqrt{8} \times 2\sqrt{8}$

Answer : i. $3\sqrt{12} \times \sqrt{18}$

$3\sqrt{12} \times \sqrt{18} = 3\sqrt{(2 \times 2 \times 3)} \times \sqrt{(2 \times 3 \times 3)}$

$\Rightarrow 3\sqrt{12} \times \sqrt{18} = 3 \times 2\sqrt{3} \times 3\sqrt{2}$

$\Rightarrow 3\sqrt{12} \times \sqrt{18} = 6\sqrt{3} \times 3\sqrt{2}$

$\Rightarrow 3\sqrt{12} \times \sqrt{18} = 18\sqrt{6}$

ii. $3\sqrt{12} \times 7\sqrt{15}$

$3\sqrt{12} \times 7\sqrt{15} = 3\sqrt{(2 \times 2 \times 3)} \times 7\sqrt{(3 \times 5)}$

$\Rightarrow 3\sqrt{12} \times 7\sqrt{15} = 3 \times 2\sqrt{3} \times 7\sqrt{(3 \times 5)}$

$\Rightarrow 3\sqrt{12} \times 7\sqrt{15} = 3 \times 2 \times 7 \times \sqrt{(3 \times 3 \times 5)}$

$\Rightarrow 3\sqrt{12} \times 7\sqrt{15} = 3 \times 2 \times 7 \times 3\sqrt{5}$

$\Rightarrow 3\sqrt{12} \times 7\sqrt{15} = 126\sqrt{5}$

iii. $3\sqrt{8} \times \sqrt{5}$

$3\sqrt{8} \times \sqrt{5} = 3\sqrt{(2 \times 2 \times 2)} \times \sqrt{5}$

$\Rightarrow 3\sqrt{8} \times \sqrt{5} = 3 \times 2\sqrt{2} \times \sqrt{5}$

$\Rightarrow 3\sqrt{8} \times \sqrt{5} = 3 \times 2 \times \sqrt{(2 \times 5)}$

$\Rightarrow 3\sqrt{8} \times \sqrt{5} = 6\sqrt{10}$

iv. $5\sqrt{8} \times 2\sqrt{8}$

$5\sqrt{8} \times 2\sqrt{8} = 5\sqrt{(2 \times 2 \times 2)} \times 2\sqrt{(2 \times 2 \times 2)}$

$\Rightarrow 5\sqrt{8} \times 2\sqrt{8} = 5 \times 2\sqrt{2} \times 2 \times 2\sqrt{2}$

$\Rightarrow 5\sqrt{8} \times 2\sqrt{8} = 5 \times 2 \times 2 \times 2 \times \sqrt{(2 \times 2)}$

$\Rightarrow 5\sqrt{8} \times 2\sqrt{8} = 5 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow 5\sqrt{8} \times 2\sqrt{8} = 80$

Q. 8. Divide, and write the answer in simplest form.

i. $\sqrt{98} \div \sqrt{2}$
ii. $\sqrt{125} \div \sqrt{50}$

iii. √54 ÷ √27
iv. √310 ÷ √5

Answer :

i. √98 ÷ √2

$$\sqrt{98} \div \sqrt{2} = \sqrt{\frac{98}{2}}$$

$$\Rightarrow \sqrt{98} \div \sqrt{2} = \sqrt{49}$$

⇒√98 ÷ √2 = 7

ii. √125 ÷ √50

$$\sqrt{125} \div \sqrt{50} = \sqrt{\frac{125}{50}}$$

$$\Rightarrow \sqrt{125} \div \sqrt{50} = \sqrt{\frac{5 \times 5 \times 5}{5 \times 2 \times 5}}$$

$$\Rightarrow \sqrt{125} \div \sqrt{50} = \sqrt{\frac{5}{2}}$$

iii. √54 ÷ √27

$$\sqrt{54} \div \sqrt{27} = \sqrt{\frac{54}{27}}$$

⇒√54 ÷ √27 = √2

iv. √310 ÷ √5

$$\sqrt{310} \div \sqrt{5} = \sqrt{\frac{310}{5}}$$

$$\Rightarrow \sqrt{310} \div \sqrt{5} = \sqrt{\frac{2 \times 5 \times 31}{5}}$$

⇒ √310 ÷ √5 = √62

Q. 9. Rationalize the denominator.

i. $\frac{3}{\sqrt{5}}$ **ii.** $\frac{1}{\sqrt{14}}$

iii. $\frac{5}{\sqrt{7}}$ **iv.** $\frac{6}{9\sqrt{3}}$

v. $\frac{11}{\sqrt{3}}$

Answer : i. We know that √5 × √5 = 5, ∴ to rationalize the denominator of $\frac{3}{\sqrt{5}}$ multiply both numerator and denominator by √5.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\Rightarrow \frac{3}{\sqrt{5}} = \frac{3}{5}\sqrt{5}$$

ii. We know that √14 × √14 = 14, ∴ to rationalize the denominator of $\frac{1}{\sqrt{14}}$multiply both numerator and denominator by √14.

$$\frac{1}{\sqrt{14}} = \frac{1}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{\sqrt{14}}{14}$$

$$\Rightarrow \frac{1}{\sqrt{14}} = \frac{1}{14}\sqrt{14}$$

iii. We know that √7 × √7 = 7, ∴ to rationalize the denominator of $\frac{5}{\sqrt{7}}$ multiply both numerator and denominator by √7.

$$\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

$$\Rightarrow \frac{5}{\sqrt{7}} = \frac{5}{7}\sqrt{7}$$

iv. We know that √3 × √3 = 3, ∴ to rationalize the denominator of $\frac{6}{9\sqrt{3}}$ multiply both numerator and denominator by √3.

$$\frac{6}{9\sqrt{3}} = \frac{6}{9\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{9 \times 3} = \frac{2\sqrt{3}}{9}$$

$$\Rightarrow \frac{6}{9\sqrt{3}} = \frac{2}{9}\sqrt{3}$$

v. We know that √3 × √3 = 3, ∴ to rationalize the denominator of $\frac{11}{\sqrt{3}}$ multiply both numerator and denominator by √3.

$$\frac{11}{\sqrt{3}} = \frac{11}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{11\sqrt{3}}{3}$$

$$\Rightarrow \frac{11}{\sqrt{3}} = \frac{11}{3}\sqrt{3}$$

Practice set 2.4

Q. 1. Multiply

i. √3(√7 – √3)
ii. (√5 – √7)√2
iii. (3√2 – √3)(4√3 – √2)

Answer :

i. √3(√7 – √3)

=√3 × √7 – √3 × √3

[∵√a(√b–√c)=√a×√b–√a×√c]

=√21 – 3

ii. $(\sqrt{5} - \sqrt{7})\sqrt{2}$

$= \sqrt{5} \times \sqrt{2} - \sqrt{7} \times \sqrt{2}$

$[\because \sqrt{a}(\sqrt{b}-\sqrt{c}) = \sqrt{a}\times\sqrt{b} - \sqrt{a}\times\sqrt{c}]$

$= \sqrt{10} - \sqrt{14}$

iii. $(3\sqrt{2} - \sqrt{3})(4\sqrt{3} - \sqrt{2})$

$= 3\sqrt{2}(4\sqrt{3} - \sqrt{2}) - \sqrt{3}(4\sqrt{3} - \sqrt{2})$

$= 3\sqrt{2}\times4\sqrt{3} - 3\sqrt{2}\times\sqrt{2} - \sqrt{3}\times4\sqrt{3} + \sqrt{3}\times\sqrt{2}$

$[\because \sqrt{a}(\sqrt{b}-\sqrt{c}) = \sqrt{a}\times\sqrt{b} - \sqrt{a}\times\sqrt{c}]$

$= 12\sqrt{6} - 3\times2 - 4\times3 + \sqrt{6}$

$= 12\sqrt{6} - 6 - 12 + \sqrt{6}$

$= 13\sqrt{6} - 18$

Q. 2. Rationalize the denominator.

i. $\frac{1}{\sqrt{7}+\sqrt{2}}$ **ii.** $\frac{3}{2\sqrt{5}-3\sqrt{2}}$

iii. $\frac{4}{7+4\sqrt{3}}$ **iv.** $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Answer : i. The rationalizing factor of $\sqrt{7} + \sqrt{2}$ is $\sqrt{7} - \sqrt{2}$. Therefore, multiply both numerator and denominator by $\sqrt{7} - \sqrt{2}$.

$$\frac{1}{\sqrt{7}+\sqrt{2}} = \frac{1}{\sqrt{7}+\sqrt{2}} \times \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{7}+\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})}$$

$$\Rightarrow \frac{1}{\sqrt{7}+\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{(\sqrt{7})^2-(\sqrt{2})^2}$$

[∵ (a-b)(a+b) = $a^2 - b^2$]

$$\Rightarrow \frac{1}{\sqrt{7}+\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{7-2}$$

$$\Rightarrow \frac{1}{\sqrt{7}+\sqrt{2}} = \frac{\sqrt{7}-\sqrt{2}}{5}$$

ii. The rationalizing factor of 2√5 – 3√2 is 2√5 + 3√2. Therefore, multiply both numerator and denominator by 2√5 + 3√2.

$$\frac{3}{2\sqrt{5}-3\sqrt{2}} = \frac{3}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$$

$$\Rightarrow \frac{3}{2\sqrt{5}-3\sqrt{2}} = \frac{3(2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})}$$

$$\Rightarrow \frac{3}{2\sqrt{5}-3\sqrt{2}} = \frac{6\sqrt{5}+9\sqrt{2}}{(2\sqrt{5})^2-(3\sqrt{2})^2}$$

[∵ (a-b)(a+b) = $a^2 - b^2$]

$$\Rightarrow \frac{3}{2\sqrt{5}-3\sqrt{2}} = \frac{6\sqrt{5}+9\sqrt{2}}{20-18}$$

$$\Rightarrow \frac{3}{2\sqrt{5}-3\sqrt{2}} = \frac{6\sqrt{5}+9\sqrt{2}}{2}$$

iii. The rationalizing factor of 7 + 4√3 is 7 – 4√3. Therefore, multiply both numerator and denominator by 7 – 4√3.

$$\frac{4}{7+4\sqrt{3}} = \frac{4}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$\Rightarrow \frac{4}{7+4\sqrt{3}} = \frac{4(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$\Rightarrow \frac{4}{7+4\sqrt{3}} = \frac{28-16\sqrt{3}}{(7)^2-(4\sqrt{3})^2}$$

[∵ (a-b)(a+b) = $a^2 - b^2$]

$$\Rightarrow \frac{4}{7+4\sqrt{3}} = \frac{28 - 16\sqrt{3}}{49-48}$$

$$\Rightarrow \frac{4}{7+4\sqrt{3}} = 28 - 16\sqrt{3}$$

iv. The rationalizing factor of √5 + √3 is √5 - √3. Therefore, multiply both numerator and denominator by √5 - √3.

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

[∵ (a-b)(a+b) = $a^2 - b^2$]

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5})^2+(\sqrt{3})^2-2(\sqrt{5})(\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

[∵ $(a-b)^2 = a^2 + b^2 - 2ab$]

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{5+3-2\sqrt{15}}{5-3}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{8-2\sqrt{15}}{2}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2[4-\sqrt{15}]}{2}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4-\sqrt{15}$$

Practice set 2.5

Q. 1. Find the value.

(i) |15 - 2|

(ii) |4 - 9|

(iii) |7| × |-4|

Answer : Absolute value describes the distance of a number on the number line from 0 without considering which direction from zero the number lies. The absolute value of a number is never negative.

Therefore,

i. |15 - 2| = |13| = 13

ii. |4 - 9| = |-5| = 5

iii. |7| × |-4| = 7 × 4 = 28

Q. 2. Solve.

i. |3x - 5| = 1

ii. |7 – 2x| = 5

iii. $\left|\frac{8-x}{2}\right| = 5$

iv. $\left|5 + \frac{x}{4}\right| = 5$

Answer : i. |3x - 5| = 1

⇒ 3x - 5 = 1 or 3x - 5 = -1

⇒ 3x = 1 + 5 or 3x = -1 + 5

⇒ 3x = 6 or 3x = 4

$\Rightarrow\ x = \frac{6}{3} \text{ or } x = \frac{4}{3}$

$\Rightarrow\ x = 2 \text{ or } x = \frac{4}{3}$

ii. |7 – 2x| = 5

⇒ 7 – 2x = 5 or 7 – 2x = -5

⇒ 2x = 7 - 5 or 2x = 7 + 5

⇒ 2x = 2 or 2x = 12

$\Rightarrow\ x = 1 \text{ or } x = \frac{12}{2}$

⇒ x = 1 or x = 6

iii. $\left|\frac{8-x}{2}\right| = 5$

$\Rightarrow \frac{8-x}{2} = 5 \text{ or } \frac{8-x}{2} = -5$

⇒ 8 – x = 2 × 5 or 8 – x = 2 × -5

⇒ 8 – x = 10 or 8 – x = -10

⇒ x = 8 – 10 or x = 8 + 10

⇒ x = -2 or x = 18

iv. $\left|5 + \frac{x}{4}\right| = 5$

$\Rightarrow 5 + \frac{x}{4} = 5 \text{ or } 5 + \frac{x}{4} = -5$

$\Rightarrow \frac{20 + x}{4} = 5 \text{ or } \frac{20 + x}{4} = -5$

⇒ 20 + x = 4 × 5 or 20 + x = 4 × -5

⇒ 20 + x = 20 or 20 + x = -20

⇒ x = 20 – 20 or x = -20 – 20

$\Rightarrow$ x = 0 or x = -40

Problem set 2

Q. 1 A. Choose the correct alternative answer for the questions given below.

i. Which one of the following is an irrational number?

A. √16/25
B. √5
C. 3/9
D. √196

Answer : An irrational number is a number that cannot be expressed as a fraction $\frac{p}{q}$ for any integers p and q ≠ 0.

$\sqrt{\frac{16}{25}} = \frac{4}{5}$ since it can be written as $\frac{p}{q}$, it is a rational number.

$\frac{3}{9} = \frac{1}{3}$ since it can be written as $\frac{p}{q}$, it is a rational number.

$\sqrt{196} = 14 = \frac{14}{1}$ since it can be written as $\frac{p}{q}$, it is a rational number.

Since √5 cannot be written as $\frac{p}{q}$ it is an irrational number

Therefore √5 is an irrational number.

Q. 1 B. Which of the following is an irrational number?

A. 0.17

B. $1.\overline{513}$

C. $0.27\overline{46}$

D. 0.101001000....

Answer : An irrational number is a number that cannot be expressed as a fraction $\frac{p}{q}$ for any integers p and q ≠ 0.

$0.17 = \frac{17}{100}$.

Since it can be written as $\frac{p}{q}$,

it is a rational number.

$1.\overline{513}$ is a rational number because it is a non-terminating but repeating decimal.

$0.27\overline{46}$ is a rational number because it is a non-terminating but repeating decimal.

0.101001000.... is an irrational number because it is a non-terminating and non-`repeating decimal.

Therefore, 0.101001000.... is an irrational number.

Q. 1 C. Decimal expansion of which of the following is non-terminating recurring?

A. 2/5
B. 3/16
C. 3/11
D. 137/25

Answer : A non-terminating recurring decimal representation means that the number will have an infinite number of digits to the right of the decimal point and those digits will repeat themselves.

$$\frac{2}{5} = 0.4$$

∵ it does not have an infinite number of digits to the right of the decimal point ∴it is not a non-terminating recurring decimal.

$$\frac{3}{16} = 0.1875$$

∵ it does not have an infinite number of digits to the right of the decimal point ∴it is not a non-terminating recurring decimal.

$$\frac{3}{11} = 0.2727 \ldots = 0.\overline{27}$$

∵ it has an infinite number of digits to the right of the decimal point which are repeating themselves ∴ it is a non-terminating recurring decimal.

$$\frac{137}{25} = 5.48$$

∵ it does not have an infinite number of digits to the right of the decimal point ∴it is not a non-terminating recurring decimal.

Therefore, $\frac{3}{11}$ is a non-terminating recurring decimal.

Q. 1 D. Every point on the number line represent, which of the following numbers?

A. Natural numbers

B. Irrational numbers

C. Rational numbers

D. Real numbers.

Answer : Every point of a number line is assumed to correspond to a real number, and every real number to a point. Therefore, Every point on the number line represent a real number.

Q. 1 E. The number 0.4 in p/q form is ………….

A. 4/9
B. 40/9
C. 3.6/9
D. 36/9

Answer :

$$0.4 = \frac{4}{10}$$

∵ the denominator of all the above options is 9 ∴ we multiply both numerator and denominator by 0.9 as 10 × 0.9 = 9

$$\Rightarrow 0.4 = \frac{4 \times 0.9}{10 \times 0.9}$$

$$\Rightarrow 0.4 = \frac{3.6}{9}$$

Q. 1 F. What is √n, if n is not a perfect square number?

A. Natural number
B. Rational number
C. Irrational number
D. Options A, B, C all are correct.

Answer : If n is not a perfect square number, then √n cannot be expressed as ratio of a and b where a and b are integers and b ≠ 0

Therefore, √n is an Irrational number

Q. 1 G. Which of the following is not a surd?

A. √**7**
B. 3√**17**
C. 3√**64**
D. √**193**

Answer :

$$\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4}$$

$$\Rightarrow \sqrt[3]{64} = \sqrt[3]{4^3}$$

$$\Rightarrow \sqrt[3]{64} = 4$$

Which is a rational number

Therefore, $\sqrt[3]{64}$ is not a surd.

Q. 1 H. What is the order of the surd $\sqrt[3]{\sqrt{5}}$?

A. 3
B. 2
C. 6
D. 5

Answer :

$$\sqrt[3]{\sqrt{5}} = \sqrt[3]{(5)^{\frac{1}{2}}}$$

$$\Rightarrow \sqrt[3]{\sqrt{5}} = \sqrt[3\times2]{5}$$

$\Rightarrow \sqrt[3]{\sqrt{5}} = \sqrt[6]{5}$

Therefore, the order of the surd $\sqrt[3]{\sqrt{5}}$ is 6.

Q. 1 I. Which one is the conjugate pair of 2√5 + √3?

A. -2√5 + √3
B. -2√5 - √3
C. 2√3 + √5
D. √3 + 2√5

Answer : A math conjugate is formed by changing the sign between two terms in a binomial. For instance, the conjugate of x + y is x - y.

Now,

2√5 + √3 = √3 + 2√5

Its conjugate pair = √3 - 2√5 = -2√5 + √3

∴ The conjugate pair of 2√5 + √3 = -2√5 + √3

Q. 1 J. The value of |12 – (13 + 7) × 4| is ....

A. -68

B. 68

C. -32

D. 32

Answer : |12 – (13 + 7) × 4| = |12 – 20 × 4| (Solving it according to BODMAS)

⇒ |12 – (13 + 7) × 4| = |12 – 80|

⇒ |12 – (13 + 7) × 4| = |-68|

⇒ |12 – (13 + 7) × 4| = 68

Q. 2. Write the following numbers in p/q form.

i. 0.555 ii. $29.\overline{568}$

iii. 9.315 315 ... iv. 357.417417...

v. $30.\overline{219}$

Answer : i.

$$0.555 = \frac{555}{1000}$$

$$\Rightarrow 0.555 = \frac{111}{200}$$

ii. Let

$$x = 29.\overline{568} = 29.568568\ldots$$

⇒ 1000x = 29568.568568......

Now,

1000x - x = 29568.568568 – 29.568568

⇒999x = 29539.0

$$\Rightarrow x = \frac{29539}{999}$$

$$\Rightarrow 29.\overline{568} = \frac{29539}{999}$$

iii. Let x = 9.315315…

⇒ 1000x = 9315.315315......

Now,

1000x - x = 9315.315315 – 9.315315

⇒999x = 9306.0

$$\Rightarrow x = \frac{9306}{999}$$

$\Rightarrow 9.315315 = \frac{29539}{999}$

iv. Let x = 357.417417…

⇒ 1000x = 357417.417417…

Now,

1000x - x = 357417.417417 – 357.417417

⇒999x = 357060.0

$\Rightarrow x = \frac{357060}{999}$

$\Rightarrow 357.417417\ldots = \frac{357060}{999}$

v. Let $x = 30.\overline{219} = 30.219219\ldots\ldots$

⇒ 1000x = 30219.219219…

Now,

1000x - x = 30219.219219 – 30.219219

⇒999x = 30189.0

$\Rightarrow x = \frac{30189}{999}$

$\Rightarrow 30.\overline{219} = \frac{30189}{999}$

Q. 3. Write the following numbers in its decimal form.

i. -5/7 **ii. 9/11**
iii. √5 **iv. 121/13**
v. 29/8

Answer : i.

$\frac{-5}{7} = -0.714287142871428 \ldots \ldots$

$\Rightarrow \frac{-5}{7} = -0.\overline{71428}$

ii.

$\frac{9}{11} = 0.818181 \ldots \ldots$

$\Rightarrow \frac{9}{11} = 0.\overline{81}$

iii. √5 = 2.236067977…….

iv.

$\frac{121}{13} = 9.307692307692307692 \ldots \ldots$

$\Rightarrow \frac{121}{13} = 9.\overline{307692}$

v.

$\frac{29}{8} = 3.625$

Q. 4. Show that 5 + √7 is an irrational number.

Answer : Let us assume that 5 + √7 is a rational number

$\therefore\ 5 + \sqrt{7} = \frac{a}{b}$

where, b≠0 and a, b are integers

$\Rightarrow \sqrt{7} = \frac{a}{b} - 5$

$\Rightarrow \sqrt{7} = \frac{a - 5b}{b}$

∵ a, b are integers ∴ a – 5b and b are also integers

$\Rightarrow \frac{a-5b}{b}$ is rational which cannot be possible ∵ $\frac{a-5b}{b} = \sqrt{7}$ which is an irrational number

∵ it is contradicting our assumption ∴ the assumption was wrong

Hence, 5 + √7 is an irrational number

Q. 5. Write the following surds in simplest form.

i. $\frac{3}{4}\sqrt{8}$ **ii.** $-\frac{5}{9}\sqrt{45}$

Answer : i.

$\frac{3}{4}\sqrt{8} = \frac{3}{4}\sqrt{2 \times 2 \times 2}$

$\Rightarrow \frac{3}{4}\sqrt{8} = \frac{3}{4} \times 2\sqrt{2}$

$\Rightarrow \frac{3}{4}\sqrt{8} = \frac{3}{2}\sqrt{2}$

ii.

$-\frac{5}{9}\sqrt{45} = -\frac{5}{9}\sqrt{3 \times 3 \times 5}$

$\Rightarrow -\frac{5}{9}\sqrt{45} = -\frac{5}{9} \times 3\sqrt{5}$

$\Rightarrow -\frac{5}{9}\sqrt{45} = -\frac{5}{3}\sqrt{5}$

Q. 6. Write the simplest form of rationalizing factor for the given surds.

i. √32 ii. √50
iii. √27 iv. 3/5√10
v. 3√72 vi. 4√11

Answer : i. √32

$\sqrt{32} = \sqrt{2 \times 2 \times 2 \times 2 \times 2}$

$\Rightarrow \sqrt{32} = 2 \times 2 \times \sqrt{2}$

$\Rightarrow \sqrt{32} = 4\sqrt{2}$

∴ Its rationalizing factor = √2

ii. √50

$\sqrt{50} = \sqrt{2 \times 5 \times 5}$

$\Rightarrow \sqrt{50} = 5\sqrt{2}$

∴ Its rationalizing factor = √2

iii. √27

$\sqrt{27} = \sqrt{3 \times 3 \times 3}$

$\Rightarrow \sqrt{27} = 3\sqrt{3}$

∴ Its rationalizing factor = √3

iv. $\frac{3}{5}\sqrt{10}$

∵ √10 cannot be further simplified

∴ Its rationalizing factor = √10

v. 3√72

$3\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3}$

$\Rightarrow 3\sqrt{72} = 2 \times 3 \times \sqrt{2}$

$\Rightarrow 3\sqrt{72} = 6\sqrt{2}$

∴ Its rationalizing factor = √2

vi. 4√11

∵ √11 cannot be further simplified

∴ Its rationalizing factor = √11

Q. 7. Simplify.

i. $\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$

ii. $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$

iii. $\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}}$

iv. $4\sqrt{12}-\sqrt{75}-7\sqrt{48}$

v. $2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}}$

Answer : i.

$\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$

$=\frac{4}{7}\sqrt{3\times7\times7}+\frac{3}{8}\sqrt{2\times2\times2\times2\times2\times2\times3}-\frac{1}{5}\sqrt{3\times5\times5}$

$=\frac{4}{7}\times7\sqrt{3}+\frac{3}{8}\times2\times2\times2\times\sqrt{3}-\frac{1}{5}\times5\sqrt{3}$

$=\frac{4}{7}\times7\sqrt{3}+\frac{3}{8}\times8\sqrt{3}-\frac{1}{5}\times5\sqrt{3}$

= $4\sqrt{3}$ + $3\sqrt{3}$ – $\sqrt{3}$

= $7\sqrt{3}$ – $\sqrt{3}$

= $6\sqrt{3}$

ii.

$5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$

$=5\sqrt{3}+2\sqrt{3\times3\times3}-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$= 5\sqrt{3} + 2 \times 3\sqrt{3} - \frac{\sqrt{3}}{3}$

$= \left(5 + 6 - \frac{1}{3}\right)\sqrt{3}$

$= \left(\frac{15 + 18 - 1}{3}\right)\sqrt{3}$

$= \frac{32}{3}\sqrt{3}$

iii.

$\sqrt{216} - 5\sqrt{6} + 2\sqrt{294} - \frac{3}{\sqrt{6}}$

$= \sqrt{6 \times 6 \times 6} - 5\sqrt{6} + 2\sqrt{2 \times 3 \times 7 \times 7} - \frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= 6\sqrt{6} - 5\sqrt{6} + 2 \times 7\sqrt{6} - \frac{3\sqrt{6}}{6}$

$= 6\sqrt{6} - 5\sqrt{6} + 14\sqrt{6} - \frac{\sqrt{6}}{2}$

$= \left(6 - 5 + 14 - \frac{1}{2}\right)\sqrt{6}$

$= \left(\frac{12 - 10 + 28 - 1}{2}\right)\sqrt{6}$

$= \frac{29}{2}\sqrt{3}$

iv.

$4\sqrt{12} - \sqrt{75} - 7\sqrt{48}$

$= 4\sqrt{2 \times 2 \times 3} - \sqrt{3 \times 5 \times 5} - 7\sqrt{2 \times 2 \times 2 \times 2 \times 3}$

$= 4 \times 2\sqrt{3} - 5\sqrt{3} - 7 \times 4\sqrt{3}$

$= 8\sqrt{3} - 5\sqrt{3} - 28\sqrt{3}$

$= (8 - 5 - 28)\sqrt{3}$

$= (8 - 5 - 28)\sqrt{3}$

$= -25\sqrt{3}$

v.

$2\sqrt{48} - \sqrt{75} - \frac{1}{\sqrt{3}}$

$= 2\sqrt{2 \times 2 \times 2 \times 2 \times 3} - \sqrt{3 \times 5 \times 5} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 2 \times 4\sqrt{3} - 5\sqrt{3} - \frac{\sqrt{3}}{3}$

$= 8\sqrt{3} - 5\sqrt{3} - \frac{\sqrt{3}}{3}$

$= (8 - 5 - \frac{1}{3})\sqrt{3}$

$= \left(\frac{24 - 15 - 1}{3}\right)\sqrt{3}$

$= \frac{8}{3}\sqrt{3}$

Q. 8. Rationalize the denominator.

i. $\frac{1}{\sqrt{5}}$ ii. $\frac{2}{3\sqrt{7}}$

iii. $\frac{1}{\sqrt{3}-\sqrt{2}}$ iv. $\frac{1}{3\sqrt{5}+2\sqrt{2}}$

v. $\frac{12}{4\sqrt{3}-\sqrt{2}}$

Answer : i.

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\Rightarrow \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$$

ii.

$$\frac{2}{3\sqrt{7}} = \frac{2}{3\sqrt{7}} \times \frac{3\sqrt{7}}{3\sqrt{7}}$$

$$\Rightarrow \frac{2}{3\sqrt{7}} = \frac{6\sqrt{7}}{(3\sqrt{7})^2}$$

$$\Rightarrow \frac{2}{3\sqrt{7}} = \frac{6\sqrt{7}}{63}$$

$$\Rightarrow \frac{2}{3\sqrt{7}} = \frac{6}{63}\sqrt{7}$$

$$\Rightarrow \frac{2}{3\sqrt{7}} = \frac{2}{21}\sqrt{7}$$

iii.

$$\frac{1}{\sqrt{3}-\sqrt{2}}=\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow\frac{1}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$\Rightarrow\frac{1}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$\Rightarrow\frac{1}{\sqrt{3}-\sqrt{2}}=\sqrt{3}+\sqrt{2}$$

iv.

$$\frac{1}{3\sqrt{5}+2\sqrt{2}}=\frac{1}{3\sqrt{5}+2\sqrt{2}}\times\frac{3\sqrt{5}-2\sqrt{2}}{3\sqrt{5}-2\sqrt{2}}$$

$$\Rightarrow\frac{1}{3\sqrt{5}+2\sqrt{2}}=\frac{3\sqrt{5}-2\sqrt{2}}{(3\sqrt{5})^2-(2\sqrt{2})^2}$$

$$\Rightarrow\frac{1}{3\sqrt{5}+2\sqrt{2}}=\frac{3\sqrt{5}-2\sqrt{2}}{45-8}$$

$$\Rightarrow\frac{1}{3\sqrt{5}+2\sqrt{2}}=\frac{3\sqrt{5}-2\sqrt{2}}{37}$$

v.

$$\frac{12}{4\sqrt{3}-\sqrt{2}}$$

$$\frac{12}{4\sqrt{3}-\sqrt{2}} \times \frac{4\sqrt{3}+\sqrt{2}}{4\sqrt{3}+\sqrt{2}}$$

$$= \frac{12(4\sqrt{3}+\sqrt{2})}{\left(4\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{12(4\sqrt{3}+\sqrt{2})}{48-2}$$

$$= \frac{12(4\sqrt{3}+\sqrt{2})}{46}$$